Reduced Order Model of Wind Turbine Wakes

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Reduced Order Model of Wind Turbine Wakes inside large Wind Farms

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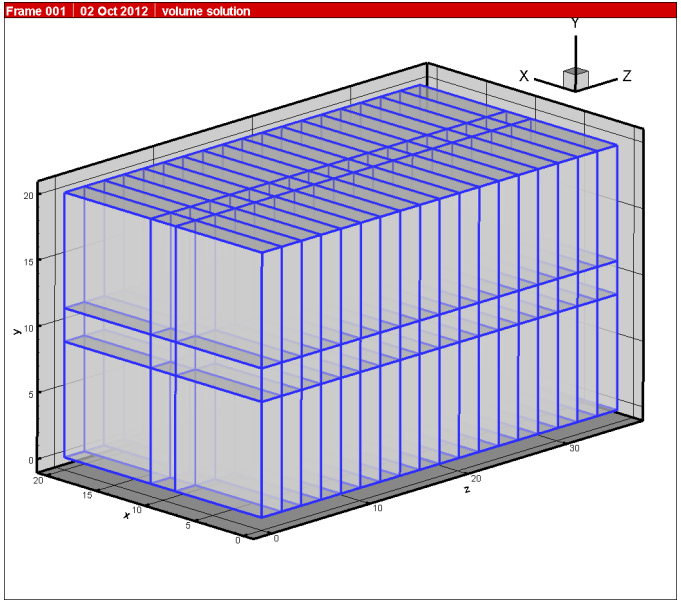
- EllipSys3D: finite volume, multi-block NS solver, parallel, non-dimensional
- LES
- Actuator Line method (Sørensen and Shen, 2002)
- NM80 Turbine with R = 40.04m
- Rated 2.75 MW @ 14 m/s
- Controller

- 20R x 20R x 36R >> 3WT, i.e. 12R spacing
- Uniform Inflow >> 15 m/s in physical space
- Cyclic BC's on inflow/outflow
- Free stream BC's (Dirichlet) on vertical and transverse



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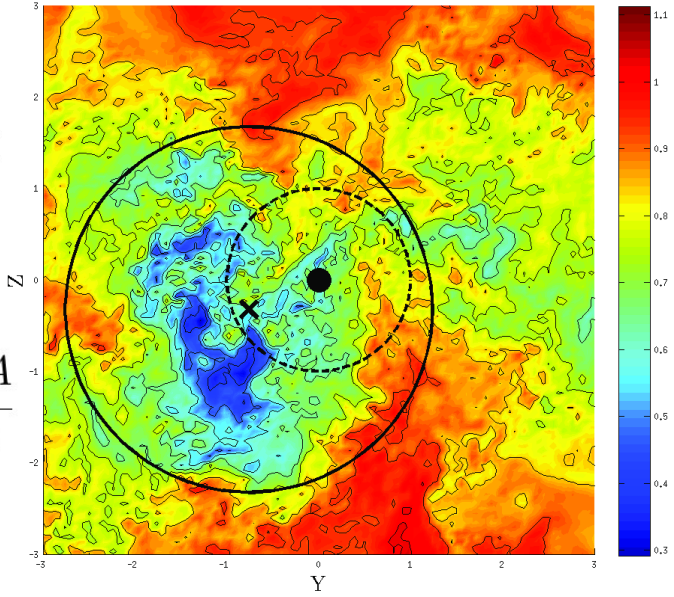
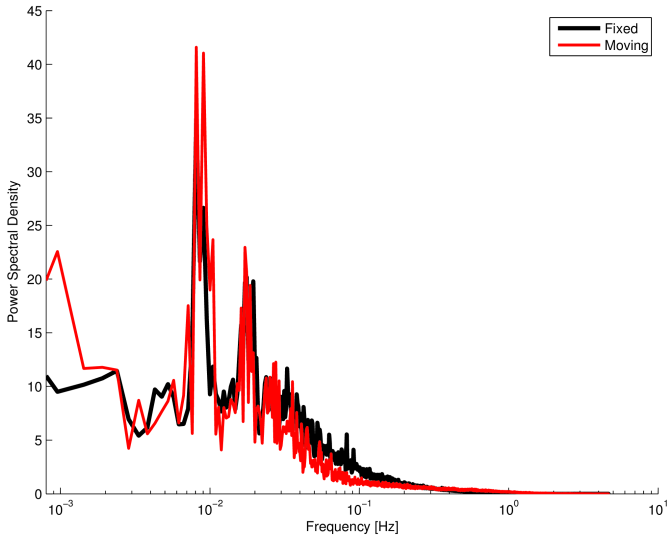
Dynamic Wake Meandering

$$y_C = \frac{\int y(1-\frac{U}{U_0})dA}{\int(1-\frac{U}{U_0})dA}$$

$$z_C = \frac{\int z(1-\frac{U}{U_0})dA}{\int(1-\frac{U}{U_0})dA}$$





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Reduced Order Model from Proper Orthogonal Decomposition (POD)

Snapshots:

$$\mathbf{U} = [\mathbf{u'_1} \dots \mathbf{u'_N}] \tag{1}$$

Auto-covariance matrix:

$$\mathbf{R} = \mathbf{U^T U} \tag{2}$$

Eigenvalue problem of the form:

$$\mathbf{RG} = \mathbf{G\Lambda} \tag{3}$$

The spatial POD modes are given as:

$$\phi_k = \frac{\mathbf{Ug_j}}{\|\mathbf{Ug_j}\|} \tag{4}$$

Reconstructing the flow using POD:

$$\mathbf{u_j} \approx \widehat{\mathbf{u}}_\mathbf{j} = \mathbf{u_0} + \sum_{k=1}^{K} \phi_k a_{kj} \tag{5}$$

where a_{kj} are the elements of $\mathbf{A} = \mathbf{\Phi^T U}$.
Truncating further to construct a Reduced Order Model:

$$\mathbf{u_j} \approx \widehat{\mathbf{u}}_\mathbf{j} \approx \widetilde{\mathbf{u}}_\mathbf{j} = \mathbf{u_0} + \sum_{k=1}^{K} \phi_k \sum_{m=1}^{M} (\beta^{(1)}_{k,m} cos(2\pi f_{k,m} t_j) - \beta^{(2)}_{k,m} sin(2\pi f_{k,m} t_j)) \tag{6}$$

The coefficients $\beta_{k,m}$ are determined by optimising the reconstruction in a linear least square sense.

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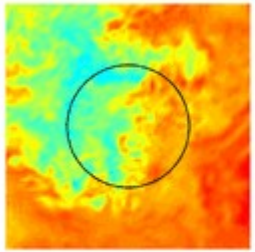
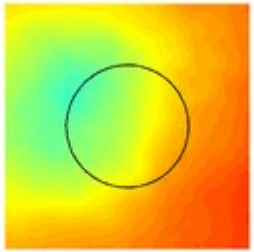
Streamwise velocity comparison

CFD



POD reconstructed
4 POD modes



Reduced Order Model
4 POD modes, 9 freq

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